Finite Math - Spring 2017
Lecture Notes - 4/24/2017

Homework

- Section 5.2 - 9, 10, 11, 12, 16, 17, 18, 19, 20, 21, 24, 33

Section 5.2 - Systems of Linear Inequalities in Two Variables

Applications.

Example 1. *A manufacturing plant makes two types of inflatable boats–a two-person boat and a four-person boat. Each two-person boat requires* 0.9 *labor-hour in the cutting department and* 0.8 *labor-hour in the assembly department. Each four-person boat requires* 1.8 *labor-hours in the cutting department and* 1.2 *labor-hours in the assembly department. The maximum labor-hours available each month in the cutting and assembly departments are* 864 *and* 672*, respectively.*

(a) *Summarize this information in a table.*

(b) *If x two-person boats and y four-person boats are manufactured each month, write a system of linear inequalities that reflects the conditions indicated. Graph the feasible region.*

Solution.

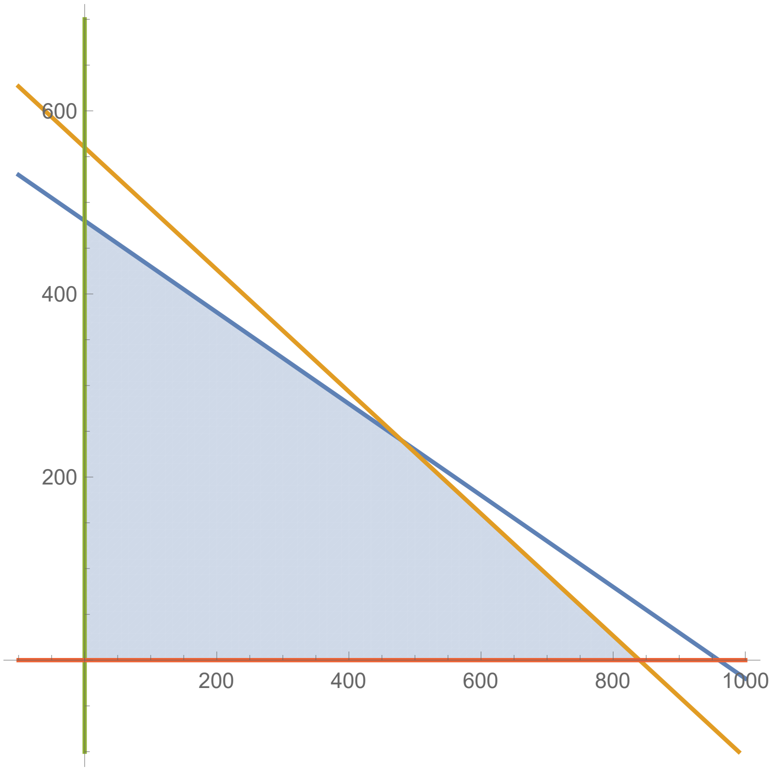
(a) *Begin by organizing the information into a table.*

	Two-Person Boat Labor-Hours	*Four-Person Boat Labor-Hours*	*Maximum Labor-Hours Available per Month*
Cutting	0.9	1.8	864
Assembly	0.8	1.2	672

(b) *The table lets us quickly come up with the system of inequalities*

$$\begin{array}{rcrcl} 0.9x & + & 1.8y & \leq & 864 \\ 0.8x & + & 1.2y & \leq & 672 \\ & & x & \geq & 0 \\ & & y & \geq & 0 \end{array}$$

Graphing all of these gives us



Example 2. *A manufacturing company makes two types of water skis, a trick ski and a slalom ski. The trick ski requires* 6 *labor-hours for fabricating and* 1 *labor-hour for finishing. The trick slalom requires* 4 *labor-hours for fabricating and* 1 *labor-hour for finishing. The maximum labor-hours available per day for fabricating and finishing are* 108 *and* 24*, respectively. If x is the number of trick skis and y is the number of slalom skis produced per day, write a system of linear inequalities that indicates appropriate restraints on x and y. Find the set of feasible solutions graphically for the number of each type of ski that can be produced.*

Section 5.3 - Linear Programming in Two Dimensions: A Geometric Approach

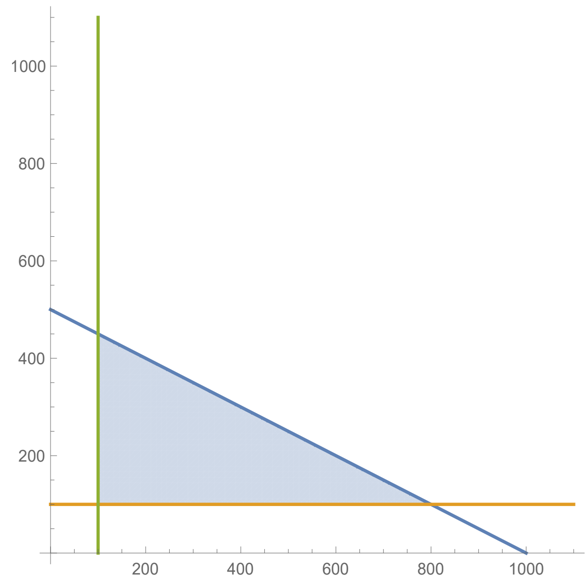
A Simple Linear Programming Problem.

Example 3. *A food vendor at a rock concert sells hot dogs for* \$4 *each and hamburgers for* \$5 *each. She purchases hot dogs for* 50¢ *each and hamburgers for* \$1 *each. If she has* \$500 *to spend on supplies, and wants to bring at least* 100 *each of hot dogs and hamburgers, how many hot dogs and hamburgers should she buy to make the most money at the concert? (Assume she sells her entire inventory.) What is her maximum revenue?*

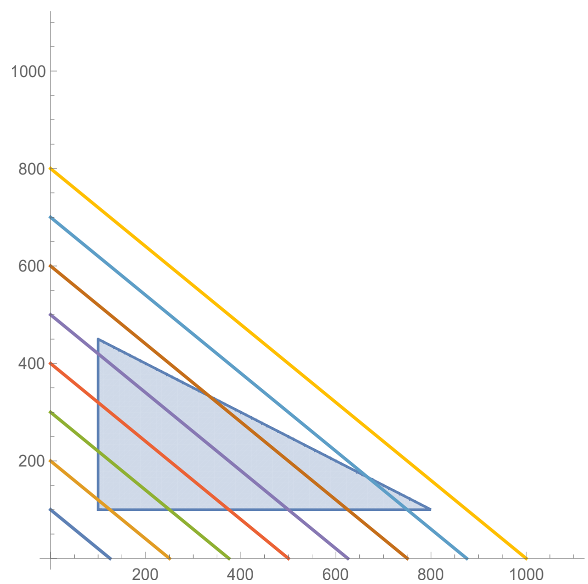
Solution. *Let x be the number of hot dogs and y be the number of hamburgers. Her revenue function is*

$$R = 4x + 5y$$

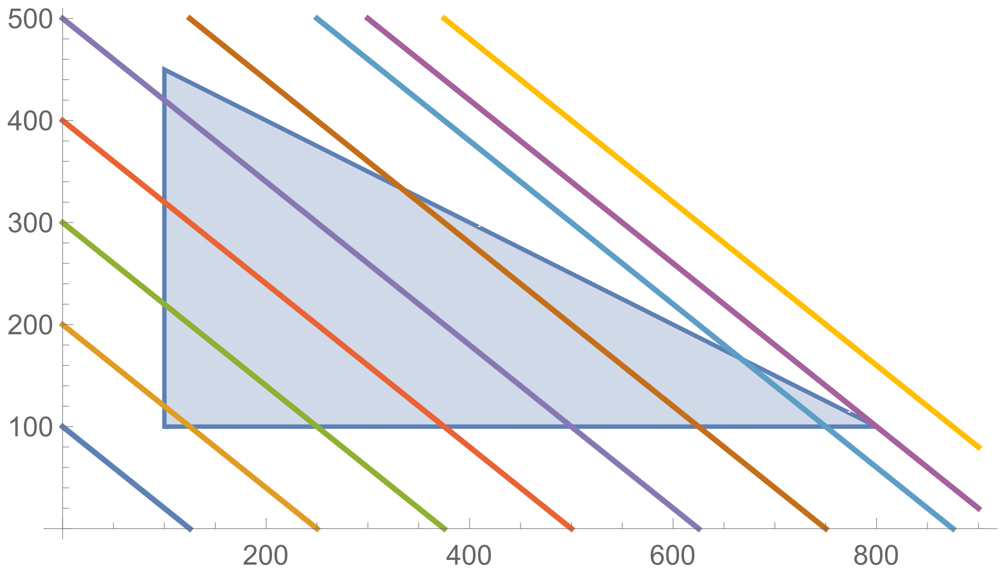
and since she is going to spend at most $\$500$ *on supplies, we get* $0.5x + y \leq 500$. *Also, since she will buy at least* 100 *each of hot dogs and hamburgers, we also have* $x \geq 100$ *and* $y \geq 100$. *Now, we graph the feasible region.*



To figure out if she can make R_0 *dollars in sales, we graph the line* $4x + 5y = R_0$ *and see if it hits the feasible region. If it does, it is possible to make that much in sales. In the next figure, we graph the revenue function for the values* $500, 1000, 1500, 2000, 2500, 3000, 3500, 4000$.



We know now that the maximum revenue is somewhere between $\$3500$ *and* $\$4000$. *If we observe how the intersection of the revenue line and the feasible region is changing, we can see that the last time the revenue line will hit the feasible region is at the corner point at the bottom right of the feasible region. Thus, the maximum revenue will happen if she sells* 800 *hot dogs and* 100 *hamburgers. Her revenue will be* $4(800) + 5(100) = 3700$ *dollars in this case. Observe if the line* $4x + 5y = 3700$ *is graphed in the above plot*



we can see that bumping that line slightly up/right will move it off of the feasible region, so the point $(800, 100)$ *must be the maximum, and the maximum revenue must be* $\$3700$.